Package 'ldsr'

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     The advantage of this method is the additional state trajectory which can reveal more information
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```

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Description

Calculate reconstruction metrics from the instrumental period

call_method 3

Usage

```
calculate_metrics(sim, obs, z, norm.fun = mean)
```

Arguments

sim A vector of reconstruction output for instrumental period

obs A vector of all observations

z A vector of left out indices in cross validation

norm. fun The function (unquoted name) used to calculate the normalizing constant. De-

fault is mean(), but other functions such as sd() can also be used. THe function must take a vector as input and return a scalar as output, and must have an argu-

ment na.rm = TRUE.

Value

A named vector of performance metrics

Examples

```
calculate_metrics(rnorm(100), rnorm(100), z = 1:10)
calculate_metrics(rnorm(100), rnorm(100), z = 1:10, norm.fun = sd)
```

call_method

Call a reconstruction method

Description

Call a reconstruction method subroutine according to the method required

Usage

```
call_method(
   y,
   u,
   v,
   method,
   init,
   num.restarts,
   return.init,
   ub,
   lb,
   num.islands,
   pop.per.island,
   niter,
   tol
)
```

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Arguments

у	Catchment output, preprocessed from data
u	Input matrix for a single-model reconstruction, or a list of input matrices for an ensemble reconstruction.
V	Same as u.
method	By default this is "EM". There are experimental methods but you should not try.
init	A list, each element is a vector of initial values for the parameters. If NULL, will be created by make_init(). See make_init for details.
num.restarts	The number of initial conditions to start the EM search; ignored if init is provided.
return.init	If TRUE, the list of initial values (init) will be returned. This can be useful if you want to reproduce the model from this one set of initial values.
ub	Upper bounds, a vector whose length is the number of parameters
1b	Lower bounds
num.islands	Number of islands (if method is GA; experimental)
pop.per.island	Initial population per island (if method is GA; experimental)
niter	Maximum number of iterations, default 1000
tol	Tolerance for likelihood convergence, default 1e-5. Note that the log-likelihood is normalized by dividing by the number of observations.

Value

The results as produced by LDS_EM_restart() when the default method (EM) is called. Other methods are experimental.

corr	Pearson's correlation	
------	-----------------------	--

Description

Calculate the Pearson's correlation using the numerically stable formulation (see References). Internal function.

Usage

```
corr(x, y)
```

Arguments

X	First variable
У	Second variable

cvLDS 5

Value

Pearson's correlation

Reference John D. Cook's article at https

//www.johndcook.com/blog/2008/11/05/how-to-calculate-pearson-correlation-accurately/

cvLDS

Cross validate LDS model. This is a wrapper for LDS_reconstruction

Description

Cross validate LDS model. This is a wrapper for LDS_reconstruction

Usage

```
cvLDS(
  Qa,
  u,
  ٧,
  start.year,
 method = "EM",
  transform = "log",
 num.restarts = 50,
  Z = make_Z(Qa$Qa),
 metric.space = "transformed",
  use.raw = FALSE,
  ub = NULL,
  1b = NULL,
  num.islands = 4,
  pop.per.island = 100,
 niter = 1000,
  tol = 1e-05
)
```

Arguments

Qa	Observations: a data.frame of annual streamflow with at least two columns: year and Qa.
u	Input matrix for a single-model reconstruction, or a list of input matrices for an ensemble reconstruction.
V	Same as u.
start.year	Starting year of the climate proxies, i.e, the first year of the paleo period. start.year + nrow(pc) - 1 will determine the last year of the study horizon, which must be greater than or equal to the last year in Qa.
method	By default this is "EM". There are experimental methods but you should not try.

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Flow transformation, either "log", "boxcox" or "none". Note that if the Box-Cox transform is used, the confidence interval after back-transformation is simply the back-transform of the trained onfidence interval; this is hackish and not entirely accurate.
The number of initial conditions to start the EM search; ignored if init is provided.
A list of cross-validation folds. If NULL, will be created with make_Z() with default settings. Users are advised to use make_Z() to create the cross-validation folds beforehand. See make_Z for details.
Either "transformed" or "original", the space to calculate the performance metrics.
Whether performance metrics are calculated on the raw time series. Experimental; don't use.
Upper bounds, a vector whose length is the number of parameters
Lower bounds
Number of islands (if method is GA; experimental)
Initial population per island (if method is GA; experimental)
Maximum number of iterations, default 1000
Tolerance for likelihood convergence, default 1e-5. Note that the log-likelihood is normalized by dividing by the number of observations.

Value

A list of cross validation results

- metrics.dist: distribution of performance metrics across all cross-validation runs; a matrix, one column for each metric, with column names.
- metrics: average performance metrics; a named vector.
- target: the (transformed) observations used for cross-validation; a data.table with two columns (year, y)
- Ycv: the predicted streamflow in each cross validation run; a matrix, one column for each cross-validation run
- Z: the cross-validation

Examples

```
# Make a shorter time horizon so that this example runs fast
u <- v <- t(NPpc[601:813])
# We run this example without parallelism.
foreach::registerDoSEQ()
cvLDS(NPannual, u, v, start.year = 1800, num.restarts = 2,
        Z = make_Z(NPannual$Qa, nRuns = 1))
# Please refer to the vignette for the full run with parallel options. It takes a minute or two.</pre>
```

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CV	Рι	$^{\circ}$	R

 $Cross\ validation\ of\ PCR\ reconstruction.$

Description

Cross validation of PCR reconstruction.

Usage

```
cvPCR(
   Qa,
   pc,
   start.year,
   transform = "log",
   Z = NULL,
   metric.space = "transformed"
)
```

Arguments

Qa	Observations: a data.frame of annual streamflow with at least two columns: year and Qa.
pc	For a single model: a data.frame, one column for each principal component. For an ensemble reconstruction: a list, each element is a data.frame of principal components.
start.year	Starting year of the climate proxies, i.e, the first year of the paleo period. start.year + nrow(pc) - 1 will determine the last year of the study horizon, which must be greater than or equal to the last year in Qa.
transform	Flow transformation, either "log", "boxcox" or "none". Note that if the Box-Cox transform is used, the confidence interval after back-transformation is simply the back-transform of the trained onfidence interval; this is hackish and not entirely accurate.
Z	A list of cross-validation folds. If NULL, will be created with make_Z() with default settings. Users are advised to use make_Z() to create the cross-validation folds beforehand. See make_Z for details.
metric.space	Either "transformed" or "original", the space to calculate the performance metrics.

Value

A list of cross validation results

- metrics.dist: distribution of performance metrics across all cross-validation runs; a matrix, one column for each metric, with column names.
- metrics: average performance metrics; a named vector.

inv_boxcox

- obs: the (transformed) observations, a data.table with two columns (year, y)
- Yev: the predicted streamflow in each cross validation run; a matrix, one column for each cross-validation run
- Z: the cross-validation fold

Examples

```
cvPCR(NPannual, NPpc, start.year = 1200)
```

exp_ci

Exponential confidence interval

Description

Get the confidence interval of $Q = \exp(Y)$ when Y is normal, i.e, Q is log-normal.

Usage

```
exp_ci(y, sigma2)
```

Arguments

y A vector of model estimates

sigma2 The variance of y as learned from a model

Value

A data.table with the updated confidence intervals

inv_boxcox

Inverse Box-Cox transform

Description

Inverse Box-Cox transform

Usage

```
inv_boxcox(x, lambda)
```

Arguments

x A numeric vector to be transformed

lambda Lambda parameter

Kalman_smoother 9

Value

The inverse Box-Cox

Examples

```
inv_boxcox(x = rnorm(10), lambda = 1)
inv_boxcox(x = rnorm(10), lambda = 0) # exp(x)
```

Kalman_smoother

Implement Kalman smoothing

Description

Estimate the hidden state and expected log-likelihood given the observations, exogeneous input and system parameters. This is an internal function and should not be called directly.

Usage

```
Kalman_smoother(y, u, v, theta, stdlik = TRUE)
```

Arguments

У	Observation matrix (may need to be normalized and centered before hand) (q rows, T columns)
u	Input matrix for the state equation (m_u rows, T columns)
V	Input matrix for the output equation (m_v rows, T columns)
theta	A list of system parameters (A, B, C, D, Q, R)'
stdlik	Boolean, whether the likelihood is divided by the number of observations. Standardizing the likelihood this way may speed up convergence in the case of long time series.

Value

A list of fitted elements (X, Y, V, J, and lik)

Note

This code only works on one dimensional state and output at the moment. Therefore, transposing is skipped, and matrix inversion is treated as /, and log(det(Sigma)) is treated as log(Sigma).

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KGE

Kling-Gupta Efficiency

Description

Kling-Gupta Efficiency

Usage

```
KGE(yhat, y)
```

Arguments

yhat Model outputs y Observations

Value

KGE

Examples

```
KGE(rnorm(100), rnorm(100))
```

LDS_BFGS

Learn LDS with L-BFGS-B

Description

Warning This is an experimental feature. Use with care.

Usage

```
LDS_BFGS(y, u, v, ub, lb, num.restarts = 100, parallel = TRUE)
```

Arguments

у	Transformed and standardized streamflow
u	Input matrix for a single-model reconstruction, or a list of input matrices for an ensemble reconstruction.
V	Same as u.
ub	Upper bounds, a vector whose length is the number of parameters
lb	Lower bounds
num.restarts	The number of initial conditions to start the EM search; ignored if init is provided.
parallel	Logical, whether parallel computation is used

Value

A list of reconstruction results; see LDS_reconstruction

Description

Warning This is an experimental feature. Use with care.

Usage

```
LDS_BFGS_with_update(
   y,
   u,
   v,
   lambda = 1,
   ub,
   lb,
   num.restarts = 100,
   parallel = TRUE
)
```

Arguments

У	Transformed and standardized streamflow	
u	Input matrix for a single-model reconstruction, or a list of input matrices for an ensemble reconstruction.	
V	Same as u.	
lambda	weight for penalty	
ub	Upper bounds, a vector whose length is the number of parameters	
1b	Lower bounds	
num.restarts	The number of initial conditions to start the EM search; ignored if init is provided.	
parallel	Logical, whether parallel computation is used	

Value

A list of reconstruction results; see LDS_reconstruction

LDS_EM

LDS_EM	Learn LDS model	

Description

Estimate the hidden state and model parameters given observations and exogenous inputs using the EM algorithm. This is the key backend routine of this package.

Usage

```
LDS_EM(y, u, v, theta0, niter = 1000L, tol = 1e-05)
```

Arguments

у	Observation matrix (may need to be normalized and centered before hand) (q rows, T columns)
u	Input matrix for the state equation (m_u rows, T columns)
v	Input matrix for the output equation (m_v rows, T columns)
theta0	A vector of initial values for the parameters
niter	Maximum number of iterations, default 1000
tol	Tolerance for likelihood convergence, default 1e-5. Note that the log-likelihood is normalized

Value

A list of model results

- theta: model parameters (A, B, C, D, Q, R, mu1, V1) resulted from Mstep
- fit: results of Estep
- liks: vector of loglikelihood over the iteration steps

Note

This code only works on one dimensional state and output at the moment. Therefore, transposing is skipped, and matrix inversion is treated as /, and log(det(Sigma)) is treated as log(Sigma).

LDS_EM_restart 13

LDS_EM_restart	Learn LDS model with multiple initial conditions	
----------------	--	--

Description

This is the backend computation for LDS_reconstruction. You should not use this directly.

Usage

```
LDS_EM_restart(y, u, v, init, niter = 1000, tol = 1e-05, return.init = TRUE)
```

Arguments

У	Observation matrix (may need to be normalized and centered before hand) (q rows, T columns)
u	Input matrix for the state equation (m_u rows, T columns)
V	Input matrix for the output equation (m_v rows, T columns)
init	A list of initial parameters for the EM search. See make_init.
niter	Maximum number of iterations, default 1000
tol	Tolerance for likelihood convergence, default 1e-5. Note that the log-likelihood is normalized by dividing by the number of observations.
return.init	Indicate whether the initial condition that results in the highest

Value

a list as produced by LDS_EM. If return.init is true, a vector of initial condition is included in the list as well.

LDS_GA

Learn a linear dynamical system using Genetic Algorithm.

Description

Warning This is an experimental feature. Use with care.

Usage

```
LDS_GA(
    y,
    u,
    v,
    lambda = 1,
    ub,
    lb,
```

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```
num.islands = 4,
pop.per.island = 100,
niter = 1000,
parallel = TRUE
)
```

Arguments

y Transformed and standardized streamflow

u Input matrix for a single-model reconstruction, or a list of input matrices for an

ensemble reconstruction.

v Same as u.

lambda weight for penalty

ub Upper bounds, a vector whose length is the number of parameters

1b Lower bounds

num. islands Number of islands (if method is GA; experimental)

pop.per.island Initial population per island (if method is GA; experimental)

niter Maximum number of iterations, default 1000 parallel Logical, whether parallel computation is used

Value

A list of reconstruction results; see LDS_reconstruction

LDS_reconstruction Learn LDS model.

Description

The initial conditions can either be randomized (specifiled by num.restarts) or provided beforehand.

Usage

```
LDS_reconstruction(
  Qa,
  u,
  v,
  start.year,
  method = "EM",
  transform = "log",
  init = NULL,
  num.restarts = 50,
  return.init = FALSE,
  ub = NULL,
  lb = NULL,
```

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```
num.islands = 4,
pop.per.island = 250,
niter = 1000,
tol = 1e-05,
return.raw = FALSE
)
```

Arguments

Observations: a data.frame of annual streamflow with at least two columns: year and Qa.
Input matrix for a single-model reconstruction, or a list of input matrices for an ensemble reconstruction.
Same as u.
Starting year of the climate proxies, i.e, the first year of the paleo period. start.year + nrow(pc) - 1 will determine the last year of the study horizon, which must be greater than or equal to the last year in Qa.
By default this is "EM". There are experimental methods but you should not try.
Flow transformation, either "log", "boxcox" or "none". Note that if the Box-Cox transform is used, the confidence interval after back-transformation is simply the back-transform of the trained onfidence interval; this is hackish and not entirely accurate.
A list, each element is a vector of initial values for the parameters. If NULL, will be created by make_init(). See make_init for details.
The number of initial conditions to start the EM search; ignored if init is provided.
If TRUE, the list of initial values (init) will be returned. This can be useful if you want to reproduce the model from this one set of initial values.
Upper bounds, a vector whose length is the number of parameters
Lower bounds
Number of islands (if method is GA; experimental)
Initial population per island (if method is GA; experimental)
Maximum number of iterations, default 1000
Tolerance for likelihood convergence, default 1e-5. Note that the log-likelihood is normalized by dividing by the number of observations.
If TRUE, state and streamflow estimates without measurement updates will be returned.

Value

A list of the following elements

- rec: reconstruction results, a data.table with the following columns
 - year: calculated from Qa and the length of u

LDS_rep

- X: the estimated hidden state
- XI, Xu: lower and upper range for the 95\
- Q: the reconstructed streamflow
- Ql, Qu: lower and upper range for the 95\
- theta: model parameters
- · lik: maximum likelihood
- init: the initial condition that resulted in the maximum likelihood (if return.init = TRUE).

Examples

```
# Make a shorter time horizon so that this example runs fast u <- v <- t(NPpc[601:813]) # We run this example without parallelism foreach::registerDoSEQ() LDS_reconstruction(NPannual, u, v, start.year = 1800, num.restarts = 1) # Please refer to the vignette for the full run with parallel options. It takes a second or two.
```

LDS_rep

Multiple LDS replicates

Description

Generate multiple stochastic time series from an LDS model. This is a convenient wrapper for one_LDS_rep.

Usage

```
LDS_rep(
  theta,
  u = NULL,
  v = NULL,
  years,
  num.reps = 100,
  mu = 0,
  exp.trans = TRUE
)
```

Arguments

theta	A list of parameters: A, B, C, D, Q, R, x0, v0
u	Input matrix for the state equation (m_u rows, T columns)
V	Input matrix for the output equation (m_v rows, T columns)
years	The years of the study horizon
num.reps	The number of stochastic replicates#'
mu	Mean of the log-transformed streamflow process
exp.trans	Whether exponential transformation back to the streamflow space is required. If TRUE, both Y and Q are returned, otherwise only Y.

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Value

Same as one_LDS_rep, but the data.table consists of multiple replicates.

Examples

```
LDS_rep(theta, t(NPpc), t(NPpc), 1200:2012, num.reps = 10, mu = mean(log(NPannual$Qa)))
```

make_init

Make initial values for parameters.

Description

If init is a vector, make it a list of one element. If init is NULL, randomize it. In this case, the function will randomize the initial value by sampling uniformly within the range for each parameters (A in [0, 1], B in [-1, 1], C in [0, 1] and D in [-1, 1]).

Usage

```
make_init(p, q, num.restarts)
```

Arguments

p Dimension of u
 q Dimension of v
 num.restarts Number of randomized initial values

Value

A list of initial conditions, each element is an object of class theta.

Examples

```
make_init(5, 5, 1)
make_init(5, 5, 2)
```

		_
ma	kρ	_/

Make cross-validation folds.

Description

Make a list of cross-validation folds. Each element of the list is a vector of the cross-validation points for one cross-validation run.

Usage

```
make_Z(obs, nRuns = 30, frac = 0.1, contiguous = TRUE)
```

Arguments

obs Vector of observations.

nRuns Number of repetitions.

frac Fraction of left-out points. For leave-one-out, use frac = 1, otherwise use any

value less than 1. Default is 0.1 (leave-10%-out).

contiguous Logical. If TRUE, the default, the left-out points are made in contiguous blocks;

otherwise, they are scattered randomly.

Value

A list of cross-validation folds

Examples

```
Z <- make_Z(NPannual, nRuns = 30, frac = 0.25, contiguous = TRUE)
```

```
metrics_with_transform
```

Transform the estimates before calculating metrics

Description

If you already ran the cross-validation on transformed output and now wanted to calculate performance on the back-transformed one, or vice-versa, you don't have to rerun the whole cross-validation, but just need to transform or back-transform the cross-validation Ycv. This function helps you do that.

Usage

```
metrics_with_transform(cv, transform, lambda = NULL)
```

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Arguments

CV	Cross-validation output as	produced by	v cvLDS or cvPCR

transform Either "log", "exp", "boxcox" or "inv_boxcox"

lambda Lambda value used in Box-Cox or inverse Box-Cox

Value

A new cv object wit hthe new metrics

Examples

```
# Cross-validate with log-transform
cv <- cvPCR(NPannual, NPpc, start.year = 1200, transform = 'log', metric.space = 'transformed')
# Calculate metrics based on back-transformed values
m <- metrics_with_transform(cv, 'exp')</pre>
```

Mstep

Maximizing expected likelihood using analytical solution

Description

Maximizing expected likelihood using analytical solution

Usage

```
Mstep(y, u, v, fit)
```

Arguments

У	Observation matrix (may need to be normalized and centered before hand) (q rows, T columns)
u	Input matrix for the state equation (m_u rows, T columns)
V	Input matrix for the output equation (m_v rows, T columns)
fit	result of Kalman_smoother

Value

An object of class theta: a list of

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neg	റര	l il	L

Calculates the negative log-likelihood

Description

Calculates the negative log-likelihood

Usage

```
negLogLik(theta, u, v, y)
```

Arguments

theta A vector of model parameters, to be converted to a list

u Input matrixv Input matrixy Observations

Value

The negative log-likelihood

NPannual

Annual streamflow at Nakhon Phanom

Description

A dataset contaning annual streamflow of the Mekong River measured at station Nakhon Phanom.

Usage

NPannual

Format

A data.table with two columns

year from 1960 to 2005

Qa mean annual streamflow, cubic meter per second

Source

Mekong River Commission

NPpc 21

NPpc

Selected principal components

Description

A dataset contaning the principal components of MADA surrounding NP

Usage

NPpc

Format

A data.table

nRMSE

Normalized root-mean-square error

Description

RMSE is normalized by the normalization constant

Usage

```
nRMSE(yhat, y, normConst)
```

Arguments

yhat Model outputs y Observations

 ${\tt normConst} \qquad \qquad {\tt The normalization \ constant}$

Value

normalized RMSE

Examples

```
x <- rnorm(100)
y <- rnorm(100)
nRMSE(x, y, sd(y))</pre>
```

one_lds_cv

NSE

Nash-Sutcliffe Efficiency

Description

Nash-Sutcliffe Efficiency

Usage

```
NSE(yhat, y)
```

Arguments

yhat Model outputs y Observations

Value

NSE

Examples

```
NSE(rnorm(100), rnorm(100))
```

one_lds_cv

One cross-validation run

Description

Make one prediction for one cross-validation run. This is a subroutine that is called by cvLDS, without any checks. You should not need to use this directly.

Usage

```
one_lds_cv(
   z,
   instPeriod,
   mu,
   y,
   u,
   v,
   method = "EM",
   num.restarts = 20,
   ub = NULL,
   lb = NULL,
   num.islands = 4,
```

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```
pop.per.island = 100,
niter = 1000,
tol = 1e-06,
use.raw = FALSE
)
```

Arguments

A vector of left-out points, indexed according to the intrumental period z indices of the instrumental period in the whole record instPeriod Mean of the observations mu Catchment output, preprocessed from data Input matrix for a single-model reconstruction, or a list of input matrices for an ensemble reconstruction. Same as u. By default this is "EM". There are experimental methods but you should not try. method The number of initial conditions to start the EM search; ignored if init is pronum.restarts vided. Upper bounds, a vector whose length is the number of parameters ub 1b Lower bounds num.islands Number of islands (if method is GA; experimental) pop.per.island Initial population per island (if method is GA; experimental) niter Maximum number of iterations, default 1000 tol Tolerance for likelihood convergence, default 1e-5. Note that the log-likelihood is normalized by dividing by the number of observations.

Whether performance metrics are calculated on the raw time series. Experimen-

Value

use.raw

A vector of prediction.

one_LDS_rep	One LDS replicate	
-------------	-------------------	--

Description

Generate a single stochastic time series from an LDS model

tal: don't use.

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Usage

```
one_LDS_rep(
  rep.num,
  theta,
  u = NULL,
  v = NULL,
  years,
  mu = 0,
  exp.trans = TRUE
)
```

Arguments

rep.num	The ID number of the replicate
theta	A list of parameters: A, B, C, D, Q, R, x0, v0
u	Input matrix for the state equation (m_u rows, T columns)
V	Input matrix for the output equation (m_v rows, T columns)
years	The years of the study horizon
mu	Mean of the log-transformed streamflow process
exp.trans	Whether exponential transformation back to the streamflow space is required. If TRUE, both Y and Q are returned, otherwise only Y.

Value

A data.table. The first column is the years of the study horizon, as supplied by year. Subsequent columns are simX, simY, and simQ which are the simulated catchment state (X), log-transformed and centralized flow (Y) and flow (Q). The last column is the replicate ID number.

Examples

```
\label{eq:loss_equation} $$ $$ Learn theta $$ one\_LDS\_rep(1, theta, t(NPpc), t(NPpc), 1200:2012, mu = mean(log(NPannual$Qa))) $$ $$ $$
```

one_pcr_cv

One cross-validation run

Description

Make one prediction for one cross-validation run. This is a subroutine to be called by other cross-validation functions.

Usage

```
one_pcr_cv(df, z)
```

Arguments

df The training data, with one column named y, the (transformed) observations.

and other columns the predictors.

z A vector of left-out points

Value

A vector of prediction.

PCR_ensemble_selection

Select the best reconstruction

Description

Select the best reconstruction from an ensemble. Experimental, API may change.

Usage

```
PCR_ensemble_selection(
   Qa,
   pc,
   start.year,
   transform = "log",
   Z = NULL,
   agg.type = c("best member", "best overall"),
   criterion = c("RE", "CE", "nRMSE", "KGE"),
   return.all.metrics = FALSE
)
```

Arguments

Qa Observations: a data frame of annual streamflow with at least two columns: year	ar
--	----

and Qa.

pc For a single model: a data frame, one column for each principal component.

For an ensemble reconstruction: a list, each element is a data.frame of principal

components.

start.year Starting year of the climate proxies, i.e, the first year of the paleo period. start.year

+ nrow(pc) - 1 will determine the last year of the study horizon, which must be

greater than or equal to the last year in Qa.

transform Flow transformation, either "log", "boxcox" or "none". Note that if the Box-Cox

transform is used, the confidence interval after back-transformation is simply the back-transform of the trained onfidence interval; this is hackish and not entirely

accurate.

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Z A list of cross-validation folds. If NULL, will be created with make_Z() with default settings. Users are advised to use make_Z() to create the cross-validation folds beforehand. See make Z for details.

agg.type Type of ensemble aggregate. There are 2 options: 'best member': the member with the best performance score is used; 'best overall': if the ensemble average

is better than the best member, it will be used, otherwise the best member will

be used.

criterion The performance criterion to be used.

return.all.metrics

Logical, if TRUE, all members' performance scores (and the ensemble average's score, if agg.type == 'best overall') are returned.

Value

A list of two elements:

- choice: The index of the selection. If the ensemble is selected, returns 0.
- cv: the cross-validation results of the choice, see cvPCR for details.
- all.metrics: all members' scores, and if agg.type == 'best overall', the ensemble average's scores as well, in the last column.

Examples

PCR_reconstruction

Principal Component Regression Reconstruction

Description

Reconstruction with principal component linear regression.

Usage

```
PCR_reconstruction(Qa, pc, start.year, transform = "log")
```

Arguments

Qa Observations: a data.frame of annual streamflow with at least two columns: year and Qa.

For a single model: a data.frame, one column for each principal component. For an ensemble reconstruction: a list, each element is a data.frame of principal components.

penalized_likelihood 27

start.year Starting year of the climate proxies, i.e, the first year of the paleo period. start.year

+ nrow(pc) - 1 will determine the last year of the study horizon, which must be

greater than or equal to the last year in Qa.

transform Flow transformation, either "log", "boxcox" or "none". Note that if the Box-Cox

transform is used, the confidence interval after back-transformation is simply the back-transform of the trained onfidence interval; this is hackish and not entirely

accurate.

Value

A list of reconstruction results, with the following elements:

For a single-model reconstruction::

- rec: reconstructed streamflow with 95% prediction interval; a data.table with four columns: year, Q, Ql (lower bound), and Qu (upper bound).
- coeffs: the regression coefficients.
- sigma: the residual standard deviation.

For an ensemble reconstruction::

- rec: the ensemble average reconstruction; a data.table with two columns: year and Q.
- ensemble: a list of ensemble members, each element is reconstructed from one element of pc and is itself a list of three elements: Q (a vector of reconstructed flow), coeffs and sigma. Note that for ensemble reconstruction, ldsr does not provide uncertainty estimates. It is up to the user to do so, for example, using ensemble spread.

Examples

```
PCR_reconstruction(NPannual, NPpc, start.year = 1200)
```

penalized_likelihood Penalized likelihood objective function

Description

Kalman_smoother returns X and likelihood. The penalized likelihood is the likelihood minus the sum-of-squares of the measurement update. This is used as the fitness function in genetic algorihm.

Usage

```
penalized_likelihood(y, u, v, theta.vec, lambda)
```

Arguments

У	Observation matrix (may need to be normalized and centered before hand) (q rows, T columns)
u	Input matrix for the state equation (m_u rows, T columns)
V	Input matrix for the output equation (m_v rows, T columns)
theta.vec	a vector of parameter elements (i.e, the vectorized version of theta in Kalman_smoother)
lambda	weight of the penalty

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Value

The penalized likelihood (a real number)

|--|

Description

This function propagates the state trajectory based on the exogenous inputs only (without measurement update), and calculates the corresponding log-likelihood

Usage

```
propagate(theta, u, v, y, stdlik = TRUE)
```

Arguments

theta	A list of system parameters (A, B, C, D, Q, R)'
u	Input matrix for the state equation (m_u rows, T columns)
V	Input matrix for the output equation (m_v rows, T columns)
у	Observations
stdlik	Boolean, whether the likelihood is divided by the number of observations. Standardizing the likelihood this way may speed up convergence in the case of long time series.

Value

A list of predictions and log-likelihood (X, Y, V, lik)

Note

This code only works on one dimensional state and output at the moment. Therefore, transposing is skipped, and matrix inversion is treated as /, and log(det(Sigma)) is treated as log(Sigma).

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RE

Reduction of Error

Description

Reduction of Error

Usage

```
RE(yhat, y, yc_bar)
```

Arguments

yhat Model outputs in the validation set

y Observations in the validation set

yc_bar Mean observations in the calibration set

Value

RE

Examples

```
x <- rnorm(100)
y <- rnorm(100)
yc_bar <- mean(x[1:50])
RE(x[51:100], y[51:100], yc_bar)</pre>
```

theta

LDS parameters

Description

Theta values for Nakhon Phanom, precomputed to use in examples.

Usage

theta

Format

A list with elements A, B, C, D, Q, R, mu1, V1

vec_to_list

vec_to_list	Converts theta from a vector (as used in GA) to list (as used in Kalman smoothing)

Description

Converts theta from a vector (as used in GA) to list (as used in Kalman smoothing)

Usage

```
vec_to_list(theta.vec, d)
```

Arguments

theta.vec a vector of parameter elements

d dimention of inputs

Value

A theta object, see make_init

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